SOLUTION.

If $OP = R$, $OA = \frac{1}{2}R$. The triangles, AOB and RBS are similar; $\therefore OB = \frac{1}{2}BR = \frac{1}{3}R$. The triangles OBC and TCS are similar and $OB = \frac{1}{3}R = \frac{1}{3}ST$; $\therefore OC = \frac{1}{3}CS = \frac{1}{4}R$. And so on.

$\therefore OP : OA : OB : OC : \ldots :: 1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \ldots .$ [*W. B. Richards.*]

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If a triangle be cut out of paper and doubled over so that the crease passes through the centre of the circumscribing circle and the angle A, the area of the triangular double-part will be

$$\tfrac{1}{2}b^2 \sin^2 C \cos C \operatorname{cosec}(2C - B) \sec(C - B);$$

the angle C being greater than B. [*Yale.*]

SOLUTION.

Call O the centre of the circumscribed circle. OA cuts BC in D. Make $\angle ADF = \angle ADC$. DF cuts AB in F. Take DE on $DC = DF$. Then DAE is the triangle required.

We know that $\angle ADE = 90° - (C - B)$, and $\angle BAD = DAE = 90° - C$; also

$$\text{area } DAE = \tfrac{1}{2}\,\frac{\overline{AD}^2 \sin DAE \sin ADE}{\sin(DAE + ADE)}.$$

But $AD = \dfrac{b \sin C}{\sin ADE}$;

$$\therefore \text{ area } DAE = \tfrac{1}{2}\,\frac{b^2 \sin^2 C \sin DAE}{\sin ADE \sin(DAE + ADE)}$$

$$= \tfrac{1}{2}b^2 \sin^2 C \cos C \sec(C - B) \operatorname{cosec}(2C - B).$$

[*T. U. Taylor.*]

EXERCISE.

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A hollow sphere, external and internal radii R and r, and specific gravity s, is partly filled with water, and floats in a pond, the water in the sphere being on a level with the surface of the pond. Find the quantity of water in the hollow sphere. [*Artemas Martin.*]